CMSC 426
Principles of Computer Security

Introduction to Cryptography (continued)

Last Class We Covered

- Introduction to crypto
  - Definitions
  - Ciphers

- Block ciphers
  - DES
  - 3DES
  - AES

Any Questions from Last Time?

Today's Topics

- Block cypher modes

- Asymmetric encryption
  - Diffie-Hellman
  - RSA
  - Math (for real this time)

Confusion and Diffusion

- Important concepts in cryptography and evaluating effectiveness

- Confusion
  - Each bit of the ciphertext should depend on several parts of the key
  - Obscures the connection between key and outcome

- Diffusion
  - If a single bit of the plaintext changes, (statistically) about half of the bits in the ciphertext should change

Modes of Operation

Modes of Operation

- Block ciphers themselves are only good for encrypting a block
  - Repeatedly applying a block cipher to larger amounts of data requires a mode of operation
  - Some modes require an Initialization Vector (IV) to get started

- Different modes of operation exist for different purposes
  - Efficiency
  - Parallel encrypt and/or decrypt
  - Encrypting a stream

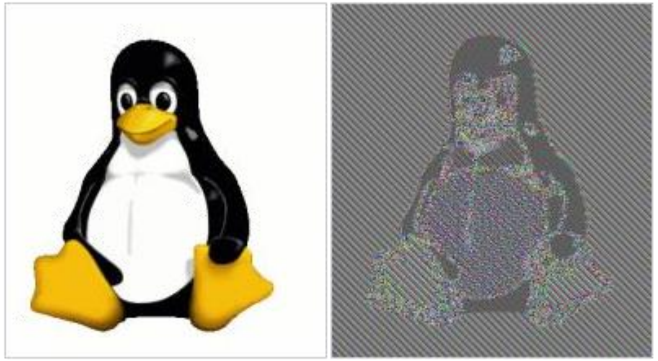
Notation

- $E_K(P)$
  - Encryption of plaintext *P* with key *K* using an arbitrary block cipher
- $D_K(C)$
  - Decryption of cipher *C* with key *K* using an arbitrary block cipher
- *Arbitrary block cipher*
  - For example, DES, 3DES, or AES

Electronic Codebook Mode (ECB)

- Simplest and most naïve mode of operation
  - Simply encrypts/decrypts each block with the same key

- Pros:
  - En/decryption can be performed in parallel
- Cons:
  - Requires padding of plaintext
  - Low diffusion



Original image Encrypted using ECB mode

$$C_i = E_K(P_i)$$

$$P_i = D_K(C_i)$$

Image taken from https://en.wikipedia.org/wiki/Block_cipher_mode_of_operation

Quick Note: Padding

- Padding involves adding garbage/filler to the end of the plaintext so that it perfectly fits within a block size

- Downside is <u>not</u> the space "wasted" on the extra text
- Rather, padding can allow an adversary to examine and learn things about the plaintext by examining the padded ciphertext
  - Not something we'll go into in depth in class
  - Read about "padding oracle attacks" for more information

Cipher Block Chaining Mode (CBC)

- Each block of plaintext is `XOR`ed with the previous ciphertext block before being encrypted
  - Uses an initialization vector for the first plaintext block

- Pros:
  - Much better diffusion
- Cons:
  - Requires padding
  - Can't parallelize encryption
    - But can parallelize decryption – why?

$$C_i = E_K(P_i \oplus C_{i-1})$$

$$P_i = D_K(C_i) \oplus C_{i-1}$$

Cipher Feedback Mode (CFB)

- Each block of plaintext is **XOR**ed with the previous ciphertext block after the previous ciphertext is re-encrypted
  - Plaintext never directly "touches" the encryption algorithm
  - Uses an initialization vector for the first plaintext block

- Block cipher is now a "stream cipher"
  - Uses the block cipher as a "key generator"
  - Digits can be encrypted one at a time, which means no padding is necessary
  - Encryption cannot be parallelized

$$C_i = E_K(C_{i-1}) \oplus P_i$$

$$P_i = E_K(C_{i-1}) \oplus C_i$$

Counter Mode (CTR)

- Also works as a stream cipher
- Requires a pseudo-random seed, *S*, to function
  - For each successive en/decrypt, the seed “counts” up by one

- Pros:
  - Encryption can be parallelized, as seed simply counts up
  - Decryption can be parallelized as well
  - Plaintext does <u>not</u> need to be padded
- Cons:
  - ???

$$C_i = E_K(S + i - 1) \oplus P_i$$

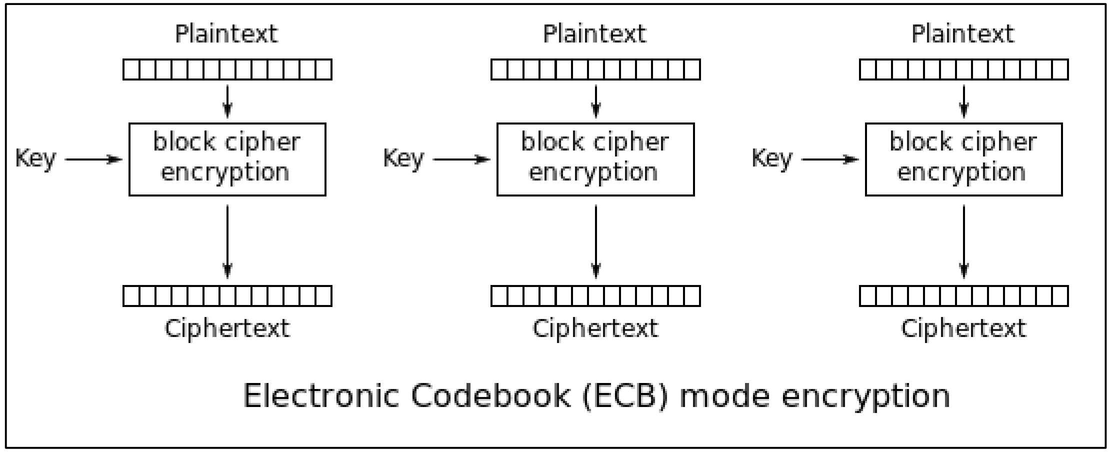
$$P_i = E_K(S + i - 1) \oplus C_i$$

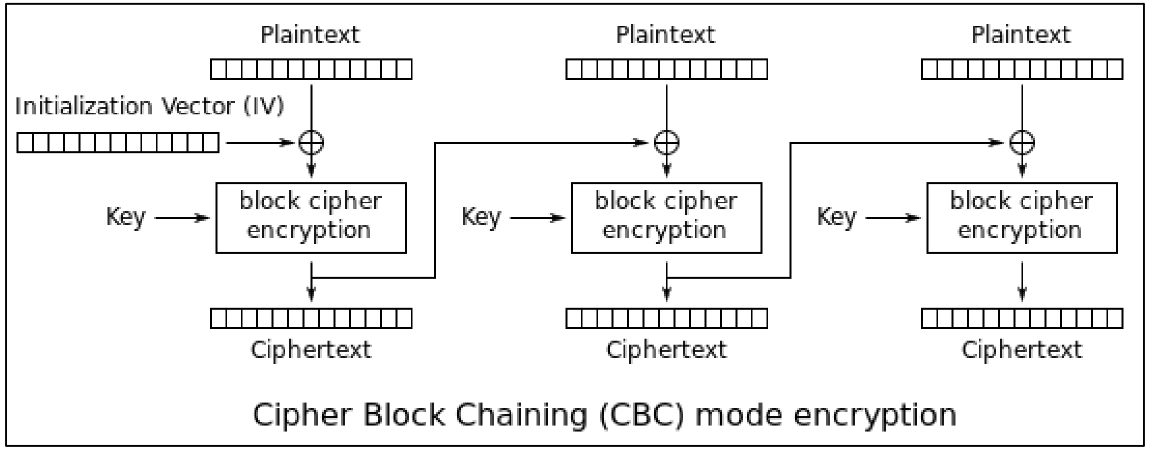
Comparison of Modes of Operation

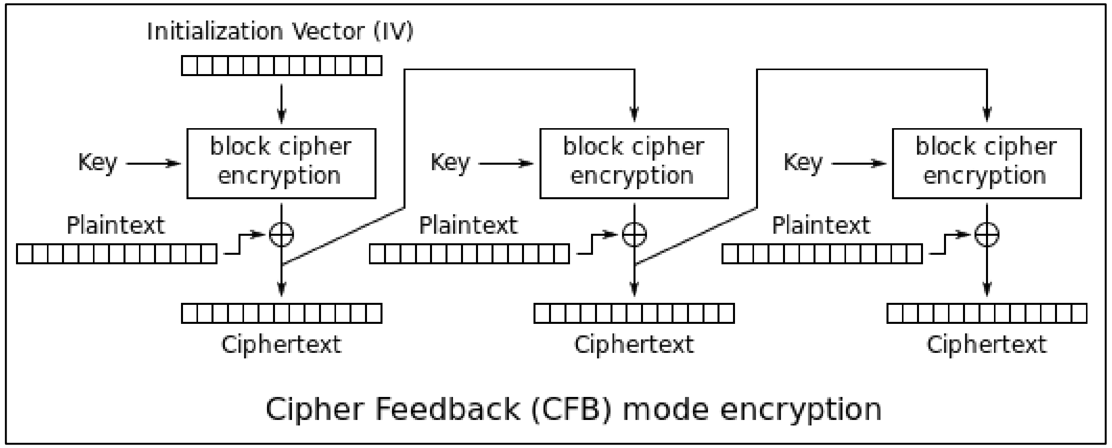
	Parallel Encrypt	Parallel Decrypt	Padding Required	Stream Cipher	Repeats in Cipher[1]
ECB	✓	✓	✓		✓
CBC		✓	✓		
CFB		✓		✓	
CTR	✓	✓		✓	

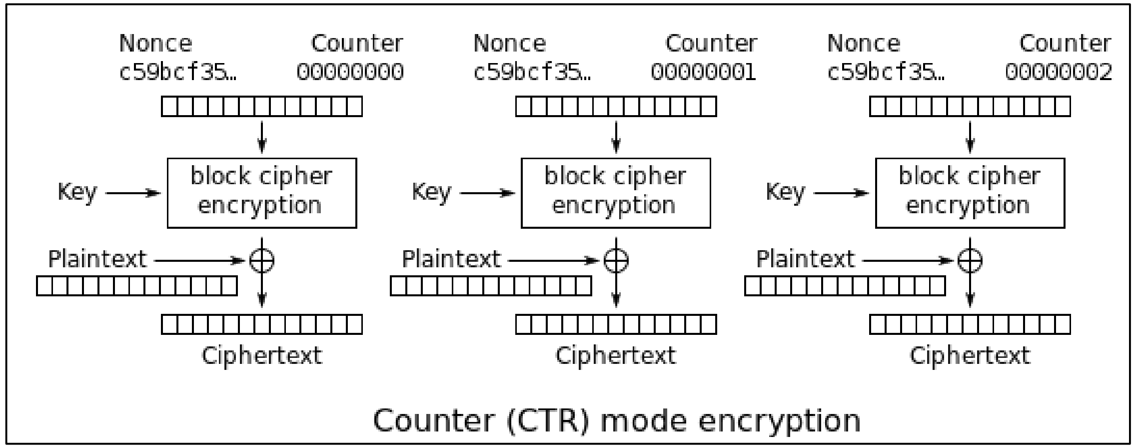
[1] Encrypting structured or repeating plaintext results in repeating cipher blocks.

Enc. Algorithms of Modes of Operation









Images taken from https://en.wikipedia.org/wiki/Block_cipher_mode_of_operation

Diffie-Hellman

Shortcomings of Symmetric Encryption

- Symmetric key must remain secret to be secure

- But how do you communicate what the secret key is?
  - Without already having a secret key?
  - ???
  - You can't!

- Need some way to share keys over an unsecured channel

Diffie-Hellman Key Exchange

- Named after Whitfield Diffie and Martin Hellman

- It is a way for two parties to
  - Use insecure communication to
  - Agree on a cryptographic key
  - Without anyone else being able to figure out what it is
- Neither party “chooses” the key, but that doesn’t matter
  - They just need the same one
- How to achieve this?
  - Math!

Basic Diffie-Hellman Algorithm

- Choose two <u>non-secret</u> values p and g
  - p is prime
  - g is generator, a primitive root modulo p (don't worry about this right now!)

- Each party
  - Chooses an integer Y in the range 1 to p - 1 (inclusive)
  - Calculates $y = g^Y \% p$ and transmit y across the <u>clear</u> channel
  - Use the <u>other</u> party's transmitted integer (x) to calculate $K = x^Y \% p$
- Both parties now have the same value K, for a symmetric key

Example Diffie-Hellman Algorithm

- Alice and Bob agree to use $p = 37$ and $g = 11$
  - Normally they would use large numbers, but this is an example

- Alice chooses the integer $A = 2$, Bob chooses $B = 9$
  - $a = g^A \% p$ $\quad a = 11^2 \% 37 \quad a = 10$
  - $b = g^B \% p$ $\quad b = 11^9 \% 37 \quad b = 36$
  - Over the clear channel, Alice transmits 10 and Bob transmits 36
- Each now calculates the key K
  - Alice: $K = b^A \% p$ $\quad K = 36^2 \% 37 \quad K = 1$
  - Bob: $K = a^B \% p$ $\quad K = 10^9 \% 37 \quad K = 1$

Diffie-Hellman: The Math

- Alice calculates $a = g^A \% p$
- Bob calculates $b = g^B \% p$
  - They transmit these values of a and b to each other, then…

- Alice calculates $K = b^A \% p$ same thing as $(g^B \% p)^A \% p$
- Bob calculates $K = a^B \% p$ same thing as $(g^A \% p)^B \% p$
  - Both of which simplify to $g^{AB} \% p$
    - (Because ~*~math~*~)

Diffie-Hellman Security

- Only p, g, a, and b are transmitted in the clear
  - So any attacker could have those

- But to calculate K, they also need either A or B
  - Which they could solve for with the formula $\log_g B \% p$
  - But this is really hard to do when p is 600 digits long
    - (For now – if this changes, we're all in deep trouble.)
- Private keys (A and B) should also be large numbers
  - Makes them difficult to calculate for an attacker, or even for the other legitimate person in the communication

RSA (not a real acronym)

RSA Overview

- RSA stands for <u>R</u>ivest, <u>S</u>hamir, and <u>A</u>dleman, its inventors
  - Is not necessarily a method for key exchange

- Is a form of asymmetric encryption
  - Uses two separate keys: public and private

- Public key is available to anyone and everyone
- Private key must be kept secret

RSA Key Generation Algorithm

- Pick two secret prime numbers, p and q
  - With those values, calculate $n = p * q$
- Choose a valid public exponent e
  - Software today uses 65537 (0x10001) to make calculations faster
    - A valid e is not a factor of n, and must be less than $(p-1)*(q-1)$ (~*~math~*~)
- Calculate a private exponent d
  - Such that e is congruent to d % (p -1) * (q -1) (more ~*~math~*~)
- Public key components are n and e
- Private key components are n and d (normally save p and q too)

Using RSA Keys

- Encryption
  - The plaintext P is converted into an integer m
    - (Don’t worry about this for now)
  - $C = m^e \% n$ (remember, e and n were our public key components)
- Decryption
  - $m = C^d \% n$ (remember, d and n were our private key components)
- Mathematical proof
  - Outside of the scope of this class (number theory, etc.)
  - Read the paper if you’re really interested

RSA Example: Key Generation

- Key generation:
  - Choose $p = 43$ and $q = 59$
  - Calculate $n = p * q$ $\quad n = 43 * 59 \quad n = 2537$
  - Choose $e = 67$
  - Calculate $d = 1927$

- Public key: $n = 2537, e = 67$
- Private key: $n = 2537, d = 1927$

RSA Example: Encryption/Decryption

- Now, someone wants to send you a message $m = 42$
  - To encrypt it, they use your public key: $n = 2537$, $e = 67$
  - $C = m^e \% n$ $\quad$ $C = 42^{67} \% 2537$ $\quad$ $C = 1332$
  - This ciphertext of 1332 is sent over a clear channel

- After receiving the message 1332, you want to read it
  - To decrypt, you'll use your private key: $n = 2537$, $d = 1927$
  - $m = C^d \% n$ $\quad$ $m = 1332^{1927} \% 2537$ $\quad$ $m = 42$

RSA Security

- An attacker has access to only *n* and *e*
  - They need access to *d* to have a complete private key
  - If they could factor *p* and *q* out of *n*, they could calculate *d*

- Fortunately, calculating the large primes that are the only factors for a large number is ***hard***
  - The larger the primes, the harder it is to factor

- Fun fact: the largest known prime is $2^{77,232,917} - 1$
  - It has 23,249,425 digits

RSA: Digital Signatures

- Encryption and decryption are inverses of each other
- If something is <u>en</u>crypted with the private key, it can be <u>de</u>crypted with the public key
  - What does this allow us to do?
  - State "only this person could have <u>en</u>crypted this"
- This is called a ***digital signature***, and is meant to prove the message came from a specific individual
  - This alone does <u>not</u> guarantee any confidentiality for the contents of the message

(Pseudo)-Random Number Generation

- `rand()` is <u>not</u> an acceptable (pseudo) random number generator for anything that has an actual purpose

- If you want something statistically viable, you need to use an actually good pseudorandom number generator (PRNG)

- If you're going to use the numbers for security-related purposes, use a **cryptographically secure pseudorandom number generator (CSPRNG)**
  - If you don't know if it's a CSPRNG, it probably isn't

Quantum Computing

- ***If*** a sufficiently large quantum computer is ever built:

- RSA and Diffie-Hellman are completely broken by an algorithm called Shor's algorithm

- The bit length of symmetric ciphers is effectively halved
  - If it would previously require 2^{128} computations to crack something, it would only require 2^{64} quantum computations

Announcements

- Lab 2 and Homework 2 are due Wednesday night
- Paper 2&3 are due next Wednesday
- Exams are graded and available for pickup